

Modelling of hand warmer

Key Stage: 3

Strand: Number and Algebra, Data Handling

Learning Units: (i) Formulae
(ii) Organisation of data
(iii) Presentation of data

Objectives: (i) state the factors to determine the *best* product
(ii) set up a mathematical formula to compare different products
(iii) learn the process of mathematical modelling

Pre-requisite Knowledge: Using charts and statistics to present the measured data

Resources Requires: hand warmers, thermometers and worksheet

Background information:

There are different brands of hand warmers. How would people choose when they purchase hand warmers? Students have to construct a model that can be used to determine the *best* hand warmer. The worksheet of the tasks can be found in the Appendix of this example.

Description of the task:

Task 1: Data collection

1. The teacher may pose the following questions to provoke students' interest in the tasks.
 - How do you choose to buy a pack of hand warmers?
 - What factors will you consider when you buy a pack of hand warmers?
 - What is the meaning of the *best* hand warmer?
2. Students are grouped into 3 or 4. They have to discuss the factors that may affect their choices. Students may write down the factors below.

_____ , _____ , _____ , _____ ,

_____ , _____ , _____ , _____ ,

_____ , _____ , _____ , _____ .

3. The teacher may continue to discuss the factors students suggested. For example, price, size, weight, appearance, duration and efficiency are some factors that may affect their choices. In order to find out the *best* hand warmer, the factors should be quantified. A score formula needs to be set to compare different hand warmers. The discussion on a score formula would be conducted in the next task.
4. The teacher may discuss with students the factor “duration” of a hand warmer. As the temperature of a hand warmer varies along time, the teacher may ask students to suggest how to define the duration of a hand warmer.

How can you define the duration of a hand warmer?

What assumption(s) are you adopting?

5. Students work in group. They have to determine at most four important factors to be included in the score formula.

Important factors are:

① _____, ② _____,

③ _____, ④ _____.

6. Afterwards, students collect data of different brands of hand warmers regarding to the factors they have chosen at home before the next task.

Measurement	Factors			
Brand	①	②	③	④
A				
B				
C				

Notes for teachers:

- Suggested factors to be considered in the task:
 - Price
 - Function / Quality
 - Duration
 - Temperature (Maximum temperature, Average temperature)
 - Size
 - Weight
 - Advertising Design
 - Effective period
 - Others
- It is expected that students may count the time for the drop of certain temperature as the duration of a hand warmer. In such case, the teacher may use the following graphs to discuss with students that other factors may be taken into consideration when measuring the duration of the hand warmer. For example, Figure 1 shows the temperature-time graphs of Brand A and Brand B hand warmers. Brand A takes 25 hours to drop the temperature from 50°C to 22°C whereas Brand B takes 25 hours to drop the temperature from 50°C to 15°C. Students have to quantitatively compare the two brands. Students may use the average temperature over certain time

intervals as a representation of duration to discriminate the two brands. One way is to calculate the mean temperature from 0 hour to 25 hours with 5 hours as the time interval.

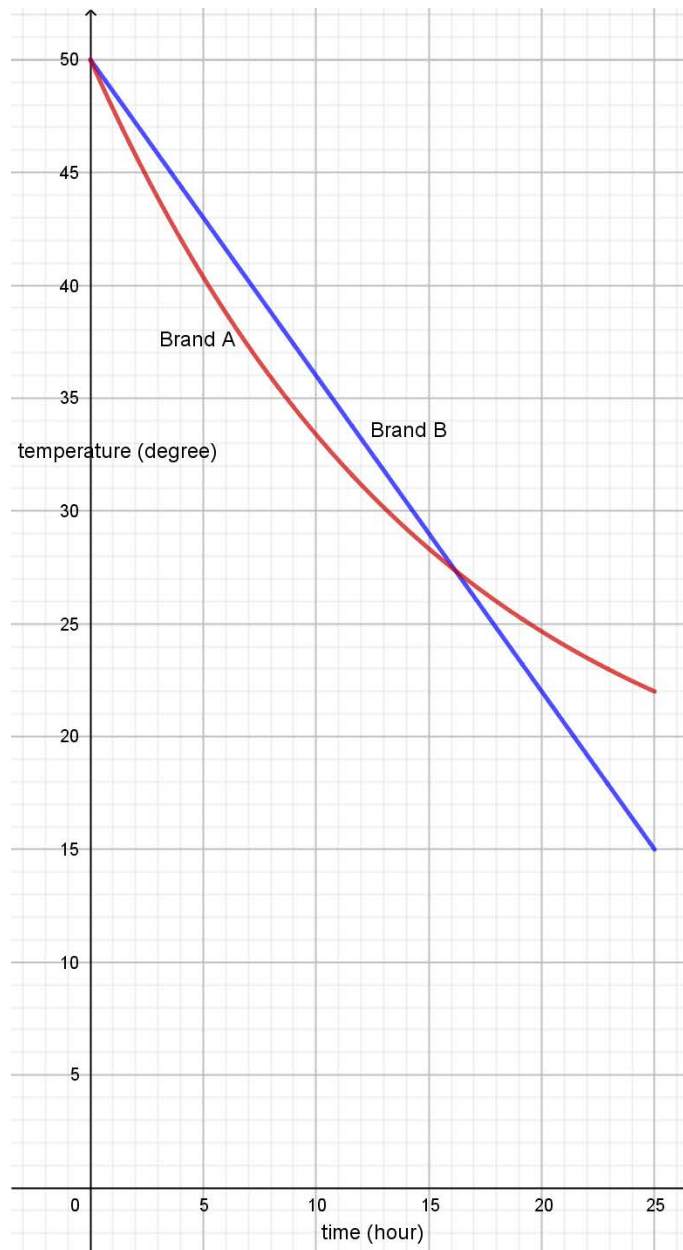


Figure 1

3. The teacher may pose the following questions to enhance the discussion on duration among students.
- How will you define the duration of a hand warmer?
 - Are the duration of Brand A and Brand B the same?
 - Can you describe the difference of these two brands of hand warmers?
 - How can you determine the average temperature of a hand warmer?

4. As students may collect the temperature data over a certain period of time. The teacher may use these real data as a starting point to introduce the concept of average temperature.
5. In case of sufficient time, students may plot the graph of temperature of a hand warmer against time. They may discuss on the features of different graphs and set appropriate criteria to determine the duration of a hand warmer.

Task 2: Mathematical modelling

1. The teacher introduces some basic concepts of mathematical modelling and let students discuss how to devise a score formula for hand warmer. The following points may be included.
 - Define some variables, measure them and make charts to represent them if necessary.
 - Formulate a score formula for comparison.
 - Communicate with other team members for discussion.
 - Review and revise the score formula if necessary.
 - Do calculations.
 - Make a final conclusion and share in the lesson.
2. The teacher may use the following formula as a starting point to let students discuss if the following score formula can be a good one.

$$\text{Score formula 1: } (\text{Duration}) \div (\text{Price})$$

Suggested questions to be discussed:

- What is the advantage of this formula?
- What is the disadvantage of this formula?
- What factors are missing?

3. The teacher may discuss with students another formula.

$$\text{Score formula 2: } 0.8 \times (\text{Duration}) \div 24 + 0.2 \times (\text{Price}) \div 5$$

Suggested questions to be discussed:

- Why is there “ $(\text{Duration}) \div 24$ ” in the formula?
- Why is there “ $0.8 \times \dots + 0.2 \times \dots$ ” in the formula?

- Students discuss in groups to devise their own score formula and input the data they collected for comparison.

<u>Our own score formula</u>
<p>What are the assumptions when you devise you own score formula?</p> <p>Assumption 1:</p> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 10px;"/> <p>Assumption 2:</p> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 10px;"/> <p>Assumption 3:</p> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 10px;"/> <p>Assumption 4:</p> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 10px;"/>

- Students list out the League table of different brands of hand warmer.

League table of different brands of hand warmer

Brand	Score	Priority
A		
B		
C		
D		

- Students present their findings to the whole class. They have to explain their assumptions in devising their own score formula. Others may comments on them.
- Students review the formula and their calculations and make modifications if necessary.

Note for Teachers:

1. Some suggested formulae for discussion on the features and shortcomings:

- ✧ $(\text{Price})+(\text{Weight})+(\text{Duration})$ ①
- ✧ $(\text{Price})\div 10+(\text{Duration})\div 12$ ②
- ✧ $0.7\times(\text{Duration})\div 24+0.3\times(\text{Price})\div 10$ ③
- ✧ $(\text{Average Temp.}-36.6)\div[(80-36.6)\div 2]$ ④

Comments on the formulae:

Formula ①: The three factors could not be added together as they come from different measurements.

Formula ②: The formula considers the factor of normalisation as it assumes the maximum price is \$10 and the maximum duration is 12 hours.

Formula ③: The formula considers the factor of normalisation as it assumes the maximum price is \$10 and the maximum duration is 24 hours. The weighted factor 0.7 and 0.3 are also included in the formula.

Formula ④: The formula assumes the minimum temperature of a hand warmer (36.6°C) as the body temperature and the maximum temperature of a hand warmer is 80°C .

2. The teacher may introduce the concept of weighted mean to students for them to set the score formulae.
3. Apart from formulating the score formula, it is important that students should be aware of the assumptions and limitations of their formulae during the process of mathematical modelling.

Worksheet

Introduction

You are going to buy a pack of hand warmers. How can you compare different brands of hand warmers? You may consider the following questions as a starting point of discussion.

- How do you choose to buy a pack of hand warmers?
- What factors will you consider when you buy a pack of hand warmers?
- What is the meaning of the *best* hand warmer?

Task 1: Data collection

1. Form a group of 3 or 4. Discuss the factors that may affect your choices. Write down the factors below.

_____ , _____ , _____ , _____ ,
 _____ , _____ , _____ , _____ ,
 _____ , _____ , _____ , _____ .

2. In order to find out the *best* hand warmer, the factors should be quantified. A score formula is needed to compare different hand warmers. The discussion on a score formula would be conducted in the next task. In this task, we are going to quantify one of the factors.
3. For example, the “duration” may be one of the major factors of a hand warmer that you consider. Answer the following questions.

How will you define the duration of a hand warmer?

What assumption(s) are you adopting?

4. Figure 1 shows the temperature-time graph of two brands of hand warmers. Brand A takes 25 hours to drop the temperature from 50°C to 22°C whereas Brand B takes 25 hours to drop the temperature from 50°C to 15°C. You have to quantitatively compare the two brands.

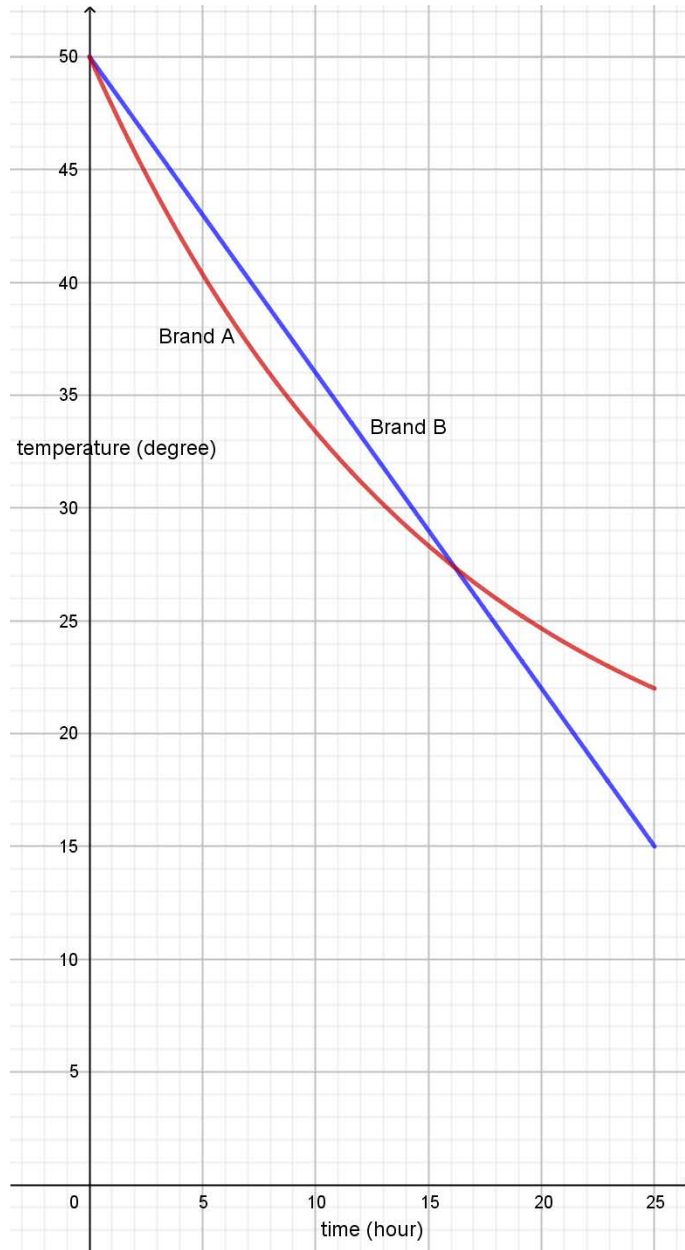


Figure 1

How will you quantitatively compare the two brands?
Write down your suggestions below.

5. Work in group. Before devising a score formula to compare different brands of hand warmers, you have to determine at most four important factors to be included in your score formula, and try to quantify each of the factors for comparison.

Important factors are:

① _____, ② _____,

③ _____, ④ _____.

How to quantify the above factors?

Factor ①

Factor ②

Factor ③

Factor ④

6. Collect data of different brands of hand warmers regarding to the factors you chosen at home before the next task.

Measurement	Factors			
Brand	①	②	③	④
A				
B				
C				

Task 2: Mathematical modelling

1. Discuss in groups to devise your own score formula and input the data you collected in Task 1 for comparison.

Our own score formula

What are the assumptions when you devise you own score formula?

Assumption 1:

Assumption 2:

Assumption 3:

Assumption 4:

2. List out the League table of different brands of hand warmer.

League table of different brands of hand warmer

Brand	Score	Priority
A		
B		
C		
D		

3. Present your findings to the whole class. Explain your assumptions in devising your own score formula.

4. Review the formula and your calculations and make modifications if necessary.

Suggested solution to the Worksheet

Task 1

1. Suggested factors to be considered:

- Price
- Function / Quality
- Duration
- Temperature (Maximum temperature, Average temperature)
- Size
- Weight
- Advertising Design
- Effective period
- Others

2. (a) Duration of a hand warmer = the time for the drop of certain temperature

- (b) Assumptions

- ✧ There are no heat loss to the surroundings
- ✧ The experiment is conducted as a fair test.

3. Taking 5 hours as the time interval,

$$\text{Average temperature of Brand A} = \frac{50+40.4+33.4+28.3+24.7+22}{6} = 33.1 \text{ (}^\circ\text{C)}$$

$$\text{Average temperature of Brand B} = \frac{50+43+36+29+22+15}{6} = 32.5 \text{ (}^\circ\text{C)}$$

Therefore, the performance of Brand A is better than that of Brand B.